LeCroy

WHITE PAPER

The Accuracy of Jitter Measurements

Introduction

Jitter is a variation in a waveform's timing. The timing variation of the period, duty cycle, frequency, etc. can be measured and compared to an average of multiple periods, or can be measured as a variation from one period to the next. A jitter measurement result will typically consist of a count of the number of periods evaluated over the selected time interval, a measurement of the highest value of jitter over that time interval, and a measurement of the standard deviation of all the jitter values over the selected time interval. In addition, various views, such as histograms, time vs. time plots, and frequency analysis of jitter, are utilized to aid in the understanding and troubleshooting of jitter.

Jitter varies randomly, or deterministically with a random component. Due to the random element in jitter measurements, individual measurements are usually of little value. Therefore, jitter is often analyzed statistically using histograms to display the distribution of many jitter measurements. When many thousands of jitter measurements are displayed in a histogram, it becomes more apparent whether the jitter is varying randomly, or whether it has a deterministic component to it. A time vs. time plot of jitter (such as LeCroy's JitterTrack™) measures the time evolution of jitter and can depict the random component of signal jitter, but is more useful in the analysis of the deterministic components of jitter.

Unlike many other measurements, there is no certified, traceable "jitter" standard. If we were measuring voltage, our equipment used to make the measurements would ultimately be traceable back to a national laboratory, such as NIST. This would ensure that the measurement we make would be the same as the meausurement made by someone else even though the equipment used was different. Without a traceable standard for jitter, it is necessary to develop an alternative method to determine the accuracy of a jitter measurement. Since much is known about the repeatability of various pieces of test equipment over a statistically significant number of periods, it is possible to develop a measurement of jitter accuracy for the standard deviation of period jitter by calculation of a confidence interval. The results obtained can be extrapolated to other types of jitter, such as cycle-cycle jitter, using multipliers. Consider the sample analysis shown in Figure 1:

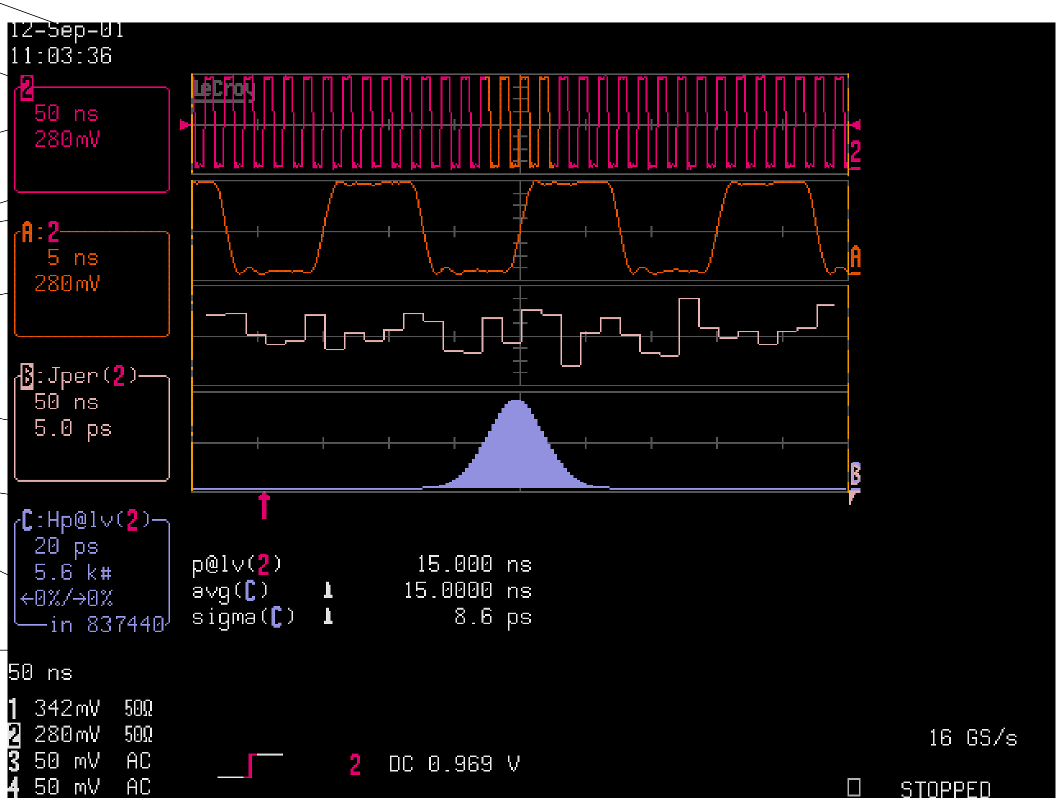

Figure 1 Jitter analysis of signal period using JitterTrack of Period at level

In this figure, a 66 MHz TTL clock signal is being analyzed using a time vs. time plot (LeCroy's JitterTrack™) and a histogram of the period jitter (p@lv, or period at level). Trace B, the JitterTrack™ of period, shows the individual measurements of variation of waveform period compared to the average period on a cycle by cycle basis. The histogram (Trace C) plots the number of period measurements as a function of the measured period. This measurement shows a typical jitter histogram with a Gaussian distribution. Distributions are classified by the shape of the histogram. Note that the histogram of period covers a horizontal range of 4 divisions, or approximately 80 ps. The vertical axis shows the number of period measurements of each incremental value. Since each measurement varies, the question of accuracy of the parameter becomes one of establishing the best estimate for the period measurements. Generally, the average or mean value is used as the estimate, or expected value, of the measured parameter. The scope's measurement of the mean value is shown as the second parameter at the bottom of the screen, avg

(C). Accuracy of the measurement system is indicated by the difference between the average value of the measured parameter, which is called the sample mean, and the true value of the parameter.

In this example, we have measured the sample mean of period and determined its value as 15.000 ps, though each individual period may differ from this mean value. The amount of variation from the sample mean, the amount of jitter in the period, is measured as 8.6 ps rms using the sigma (standard deviation) of the distribution. In order to determine the accuracy of these measurements we need a method of determining if the true period is 15.000 ns or that the true amount of period jitter is 8.6 ps rms.

The concept of precision, or measurement repeatability, is related to how closely the measured parameter values are distributed about the mean value. The dispersion of measured values is described by the standard deviation (sigma) of the histogram. The lower the standard deviation, the closer the measured values are centered about the mean. The physical interpretation of the standard deviation (sigma) for Gaussian distributed data as shown here, is that 67% of the values in the histogram are contained within ±1 standard deviation of the mean value and 99% of all values are contained within ±3 standard deviation limits.

Bear in mind that jitter is a function of the dispersion of the measured parameter. For example, root mean square (rms) jitter is defined as the standard deviation of the period at level (p@lvl) parameter. Therefore, the distribution of the jitter value is not the same as the distribution of the source parameter. In the discussion which follows, rms jitter will be measured by determining the standard deviation of the period at level parameter over a large number of cycles. Multiple rms jitter values will be acquired and these form another distribution, which has its own characteristic distribution, mean, and standard deviation. It is these values that will be used to determine the accuracy of the jitter measurement.

Specification of Jitter Accuracy as a Confidence Interval

The specification of jitter repeatability and accuracy require a probabilistic approach. Therefore, we'll measure jitter by determining how close the individual parameter measurements are to the estimate of the actual value of the parameter being measured. In equation form we would like to know how far the measured parameter, x, is from the mean value, μ, of the parameter's true distribution. The displacement is measured in terms of multiples, k, of standard deviation, σ, of the distribution, which for this discussion is assumed Gaussian.

$$\mu - k\sigma < x < \mu + k\sigma$$

Since in most cases we do not know the characteristics of the underlying parameter's statistical distribution, we must turn this statement around and ask, "How close is the mean value to the measured values?" This takes the form:

$$x - k\sigma < \mu < x + k\sigma$$

These statements are mathematically equivalent.

These statements have been based on a single measurement. If we make N repeated measurements obtaining a sample mean value, $\bar{x}$, this can be restated as follows:

$$\mu - k\frac{\sigma}{\sqrt{N}} < \bar{x} < \mu + k\frac{\sigma}{\sqrt{N}}$$

This can be reformulated to reflect on the location of the mean relative to the sample mean, $\bar{x}$.

$$\bar{x} - k\frac{\sigma}{\sqrt{N}} < \mu < \bar{x} + k\frac{\sigma}{\sqrt{N}}$$

This assumes knowledge of the standard deviation, σ. In the most general case this may not be known. We can express this equation in terms of the known sample mean, $\bar{x}$, and the standard deviation of the sample, s:

$$\bar{x} - k'\frac{s}{\sqrt{N}} < \mu < \bar{x} + k'\frac{s}{\sqrt{N}}$$

This equation states that the mean value, μ, of the underlying distribution is within $\pm k'\frac{s}{\sqrt{N}}$ of the mean value of the measured samples. This can be stated as:

$$|\bar{x} - \mu| < k'\frac{s}{\sqrt{N}}$$

or

$$\frac{|\bar{x} - \mu|}{s/\sqrt{N}} < k'$$

The term k' is associated with a new distribution function, called the Students t distribution. This distribution is a function of the sample mean, standard deviation, and number of samples. We can substitute the letter t for k' and rewrite our equation:

$$\bar{x} - t\frac{s}{\sqrt{N}} < \mu < \bar{x} + t\frac{s}{\sqrt{N}}$$

	Average Period (ns)	Standard Deviation / RMS jitter (ps)
	7.519	13.5
	7.519	13.5
	7.519	13.5
	7.519	13.4
	7.519	13.4
	7.519	13.4
	7.519	13.4
	7.519	13.5
	7.519	13.3
	7.519	13.4
	7.519	13.3
	7.519	13.2
	7.519	13.4
	7.519	13.4
	7.519	13.4
	7.519	13.4
	7.519	13.4
	7.519	13.4
	7.519	13.4
	7.519	13.3
Average	7.519	13.395
Std Dev	0	0.075915465

Table 1

We can state that the mean of the underlying distribution lies within a confidence interval of:

$$\mu < \bar{x} \pm t\frac{s}{\sqrt{N}}$$

with a certain level of probability, called the confidence coefficient, usually expressed as a percentage (e.g. 95%). The value of t is obtained from a lookup table based on the degrees of freedom (N-1), and the level of probability desired.

For example: 20 repeated measurements of the rms period jitter of a 133 MHz sine wave, made over 13298 cycles each, yield the values, shown in Table 1.

We can determine the 99% confidence interval for the rms jitter using the equation above.

$$\mu < \bar{x} \pm t\frac{s}{\sqrt{N}}$$

The value of t for a 99% confidence interval and N=20 (degrees of freedom = 20-1=19) is read from a table of the Student t distribution as 2.861. To calculate the confidence interval for the rms jitter we use the average or mean value of the rms jitter of 13.395 ps and the sample standard deviation of 0.0759 ps which yields:

$$13.346444 < \mu < 13.443556$$

this is 13.395 ±0.04856 ps (99% confidence)

So we can say with 99% confidence that the true RMS jitter of the period of this 133 MHz sinewave is within 48.56 fs of the mean of the 20 RMS jitter values measured.

This analysis has concerned itself with the confidence interval for rms period jitter. This is different from, but related to, the confidence interval for the period at level parameter. You can also calculate the confidence interval of the mean of the period measurement for a confidence coefficient of 99%.

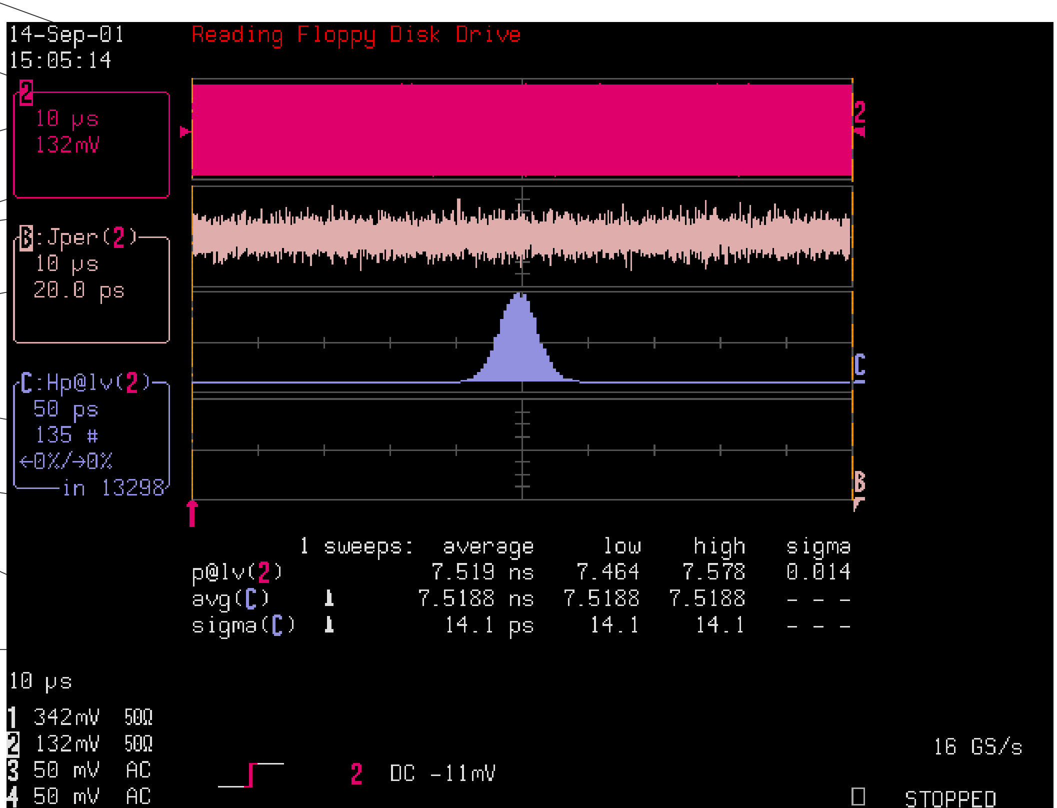

Figure 2 A single measurement acquisition of period jitter

In this case the number of samples contributing to the mean is 13,298, the sample mean period is 7.5188 ns and the sample standard deviation is 14.1 ps.

The value of t for a 99% confidence interval and N=13298 (degrees of freedom =13297 which is interpreted as infinite) is read from a table of the Student t distribution as 2.576. The 99% confidence level is :

$$7518.485 \text{ ps} < \mu < 7519.115 \text{ ps}$$

$$\text{or } 7.5188 \text{ ns} \pm 0.315 \text{ps}$$

A 10 digit frequency counter confirmed the period as 7.518796 ns which is certainly within the confidence interval.

In a similar way, using a reference source of known period, we can use the confidence interval concept to determine the accuracy specification for the measurement system. In our case we have selected an industry standard signal source, the Agilent 8133A, which has a typical rms jitter of 1 ps. Coupling this generator with a high resolution frequency counter, the period at level parameter will be used to verify the underlying accuracy of the jitter measurement. Additional studies of the derived jitter parameters such as rms period jitter will establish the resolution and repeatability of those measurements.

Table 2 shows a record of period at level measurements over a range of common clock frequencies from 33.6 MHz to 750 MHz. For each frequency, the period of the input signal was measured using a 10 digit frequency counter as well as the period at level parameter. Based on the sample mean and standard deviation a confidence interval was established for 99% probability. The confidence interval widths, an indication of the repeatability of the measurement over all frequencies, are below 0.212 ps. Note also, that the differences between the measured period, using the counter and the period at level parameter (bias of the mean), are below 0.16 ps. The conclusion to be drawn from this data is that the period at level measurement can be made to well within 1 ps accuracy over a range of frequencies.

Input Frequency MHz	Counter Period (ns)	Average p@lv (ns)	Bias (ps)	sigma (ps)	Confidence Coefficient %	Lower confidence limit (ns)	Upper confidence limit (ns)	Half width of confidence interval (ps)
33.6	29.73466	29.7345	0.16	4.7	99	29.7344144	29.7345856	0.08561319
50	19.99315	19.9931	0.05	2.19	99	19.9930601	19.9931399	0.0398921
66.7	14.98185	14.9819	-0.05	2.21	99	14.9818597	14.9819403	0.04025642
75	13.32688	13.3268	0.08	5.11	99	13.3267069	13.3268931	0.09308158
100	9.99479	9.9948	-0.01	2.16	99	9.99476065	9.99483935	0.03934564
133	7.51228	7.51226	0.02	5.32	99	7.51216309	7.51235691	0.09690685
150	6.66251	6.66249	0.02	5.24	99	6.66239455	6.66258545	0.0954496
175	5.71212	5.71215	-0.03	4.76	99	5.71206329	5.71223671	0.08670613
200	4.99942	4.99944	-0.02	2.22	99	4.99939956	4.99948044	0.04043857
233	4.29189	4.29187	0.02	4.81	99	4.29178238	4.29195762	0.0876169
250	3.99811	3.99802	0.09	2.06	99	3.99798248	3.99805752	0.03752408
275	3.63347	3.63344	0.03	4.88	99	3.63335111	3.63352889	0.08889199
300	3.33019	3.33018	0.01	4.9	99	3.33009074	3.33026926	0.08925631
333	3.00159	3.00159	0	2.25	99	3.00154901	3.00163099	0.04098504
350	2.85582	2.85581	0.01	3.86	99	2.85573969	2.85588031	0.07031211
375	2.66568	2.66567	0.01	5.11	99	2.66557692	2.66576308	0.09308158
400	2.49952	2.49951	0.01	4.94	99	2.49942002	2.49959998	0.08998493
500	1.99879	1.99877	0.02	2.09	99	1.99873193	1.99880807	0.03807055
750	1.33284	1.33284	0	5.83	99	1.3327338	1.3329462	0.10619679

Table 2

Number of samples	Confidence Coefficient (%)	Sample Mean (ps)	Sample Standard Deviation (ps)	Lower Confidence Limit (ps)	Upper Confidence Limit (ps)	Width of Confidence Interval (ps)	Freq (MHz)
20	99	2.4365	0.05993637	2.398157129	2.474842871	0.076685742	33.6
20	99	2.8315	0.07875846	2.781116144	2.881883856	0.100767712	33.6
20	99	2.6645	0.03953346	2.639209404	2.689790596	0.050581191	33.6
20	99	3.072	0.07288058	3.025376377	3.118623623	0.093247245	33.6
80	99	2.751125	0.24222071	2.679644465	2.822605535	0.142961071	33.6
20	99	2.675	0.04729527	2.644743976	2.705256024	0.060512048	50
20	99	2.2385	0.05896252	2.200780125	2.276219875	0.07543975	50
20	99	2.2245	0.03347819	2.203083119	2.245916881	0.042833761	50
20	99	2.3925	0.04677775	2.362575044	2.422424956	0.059849912	50
80	99	2.382625	0.18821442	2.327081988	2.438168012	0.111086023	50
20	99	2.7145	0.04839258	2.683541993	2.745458007	0.061916015	66.5
20	99	2.358	0.02566997	2.341578245	2.374421755	0.032843509	66.5
20	99	2.4065	0.03558163	2.383737494	2.429262506	0.045525012	66.5
20	99	2.565	0.04442617	2.536579414	2.593420586	0.056841172	66.5
80	99	2.511	0.14633584	2.467815566	2.554184434	0.086368867	66.5
20	99	2.818	0.04149826	2.791452475	2.844547525	0.05309505	106.3
20	99	2.9785	0.06846244	2.93470278	3.02229722	0.087594441	106.3
20	99	3.038	0.03349784	3.016570552	3.059429448	0.042858897	106.3
20	99	2.8245	0.03300319	2.803386992	2.845613008	0.042226016	106.3
80	99	2.91475	0.1066756	2.8832695	2.9462305	0.062961	106.3
20	99	2.6345	0.02502104	2.618493381	2.650506619	0.032013239	266
20	99	2.826	0.01500877	2.816398496	2.835601504	0.019203008	266
20	99	2.559	0.01333772	2.550467511	2.567532489	0.017064978	266
20	99	3.104	0.0218608	3.090015069	3.117984931	0.027969861	266
80	99	2.780875	0.21259812	2.718136238	2.843613762	0.125477524	266
400	99	2.668075	0.26737758	2.633473618	2.702676382	0.069202763	

Table 3

Using the same confident interval analysis we can determine the repeatability of a derived jitter measurement such as rms period jitter. Table 3 summarizes rms period jitter measurements made using four different units at 5 commonly used clock frequencies. Confidence intervals for each unit at each frequency as well as those for all units at each frequency, and all units at all frequencies have be calculated.

Note that the widest confidence limit is 0.143 ps for over 400 measurements.

The use of confidence intervals can be applied to any series of related measurements as a tool to determine the repeatability of the measurement process. So it can be used to assess any jitter parameter. These results have been obtained using a pulse source with fast edges (typically < 60 ps). Results for other wave shapes will vary. For instance, those for a sine wave will generally be greater with a decided dependence on the signal frequency (confident interval width will increase for decreasing frequency).

Jitter Distributions

One of the initial assumptions is that the distribution of the measured values is Gaussian. In figure 3 we demonstrate that even if the measured distribution is deterministic, in this case sinusoidal, the distribution of RMS jitter taken over many acquisitions is Gaussian.

In this example, the average period (15.150 ns) varies and the variation in period has a sinusoidal shape. Our fundamental period is being modulated. The rms jitter of the period is measured to be 326 ps. If we measure the jitter many times we will get slightly different values. We have made 3,655 measurements and depict their distribution in the Gaussian shaped histogram in the lower trace.

This effect is a confirmation the central limit theory of statistics which states that random samples of size N taken from a population of mean, μ, and standard deviation ,α, have a sample mean, x , which approaches the normal or Gaussian distribution as N becomes large. So that the accuracy specifications based on rms jitter measurements can always make use of confidence intervals. Figure 3 An investigation of the distribution of RMS period jitter derived from a jitter source with a deterministic distribution (sinusoidal)

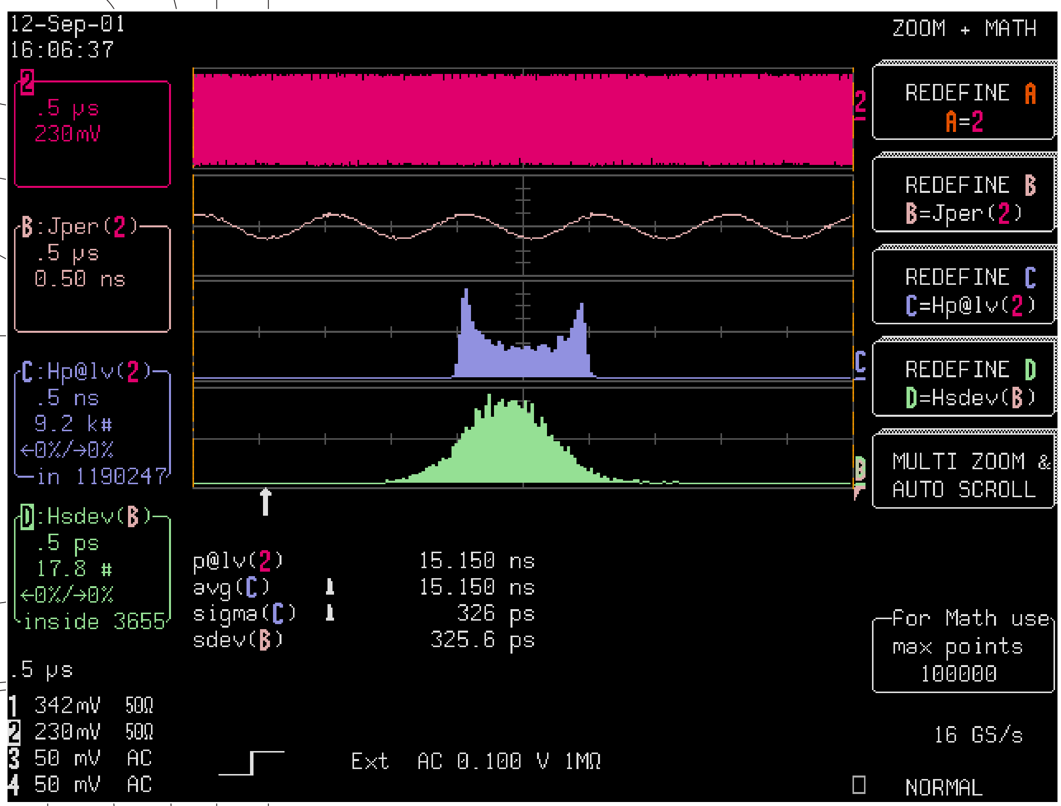

Figure 3